Concordance Correlation Coefficient & Macro

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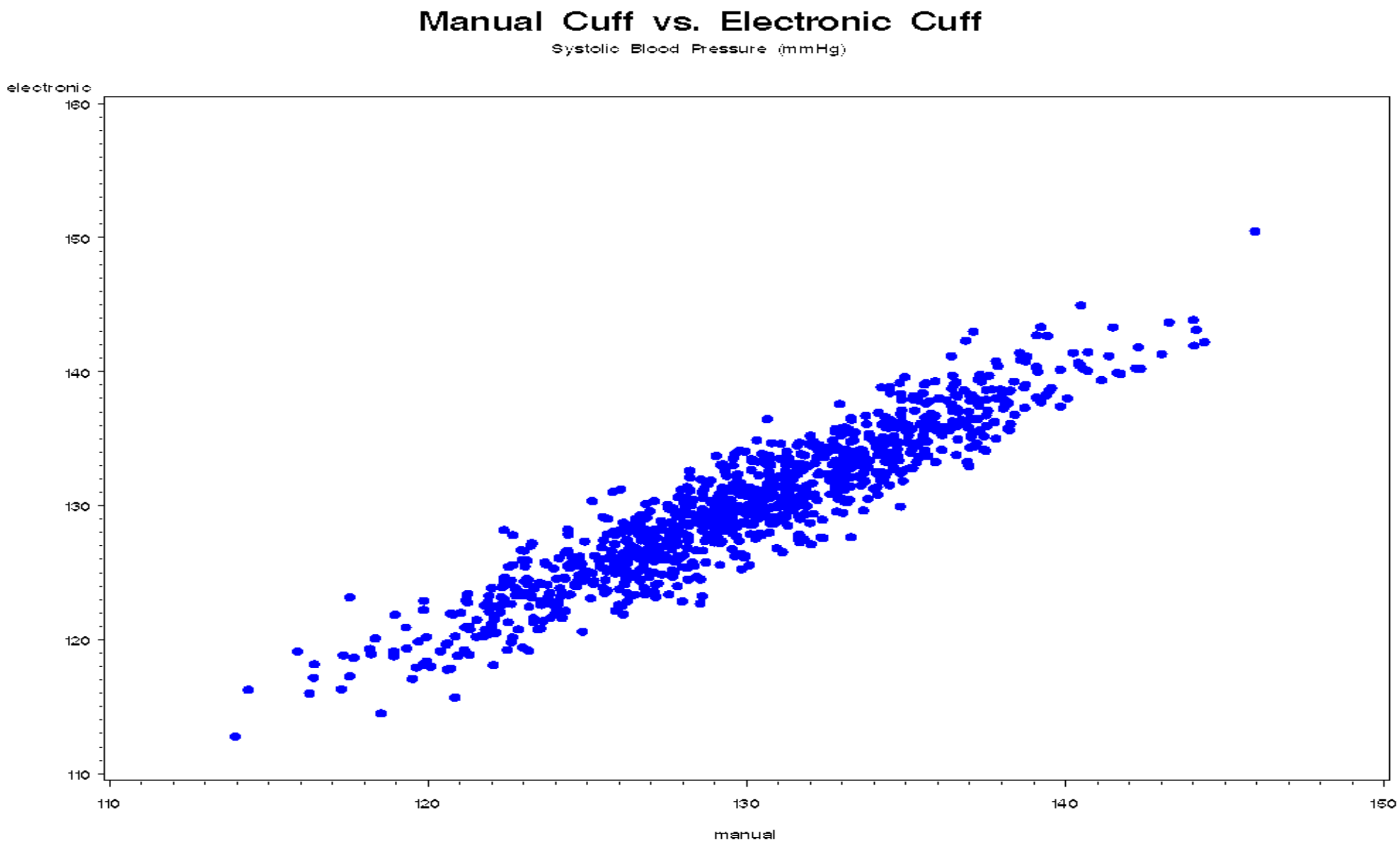
Outline

- Example
- Ways to assess agreement
- Definition of concordance correlation coefficient
- linscon.sas macro input and output
- References

Example

- Manual blood pressure cuff is the “gold standard”
- Can using the manual cuff be replaced by using a cheaper electronic cuff?
- The answer is “Yes” if the pairs (of readings from the cuffs on the same individual) are tightly packed around the 45 degree line

Example (cont.)



Ways to Assess Agreement

- Many ways exist to assess agreement
  - compare definitions of items
  - compare counts of cases in particular classes
  - Pearson correlation coefficient
  - paired t-test
  - linear regression: test if intercept = 0 and slope = 1

- These ways are OK, but they don't fully test for agreement

Ways to Assess Agreement (cont.)

- Pearson correlation coefficient, r
  - (x_i, $y_i = x_i + 1000$)
  - $\hat{\rho} = 1$, but it would not be wise to replace x by y
  - ρ only measures precision, not accuracy

- Paired t-test

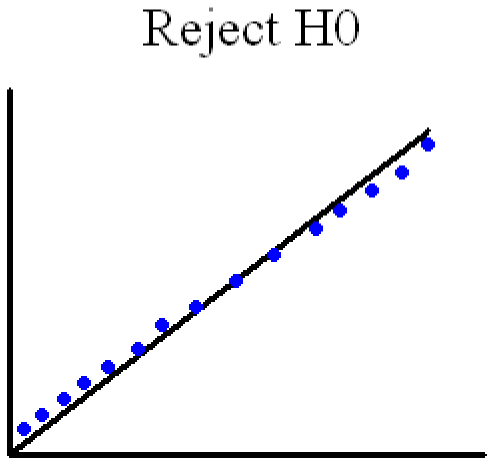
$$\begin{cases} H_0 : \mu_x = \mu_y \\ H_1 : \mu_x \neq \mu_y \end{cases}$$

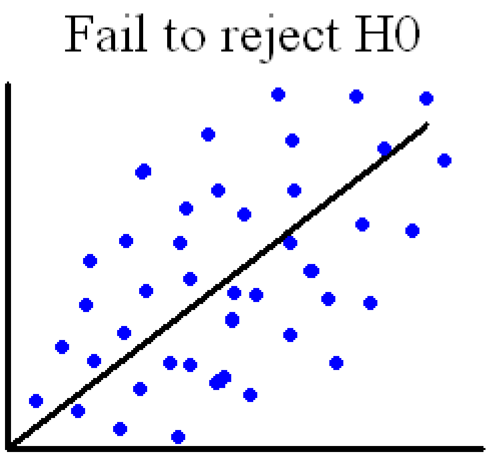
  - { (1,3), (2,3), (3,3), (4,3), (5,3) }
  - fail to reject H_0, but it would not be wise to replace x by y

Ways to Assess Agreement (cont.)

- Linear regression
  - agreement would be points lying on line going through origin at 45 degree angle (the Y=X line)
  - linear regression can fail depending on the amount of scatter

Ways to Assess Agreement (cont.)





H_0: $b_0 = 0$ and $b_1 = 1$

Concordance Correlation Coefficient

- The concordance correlation coefficient r_c is
  - the correlation between two variables that fall on the 45 degree line through the origin
  - a product of
    - precision (Pearson correlation coefficient, r) <u>and</u>
    - accuracy (closeness to 45 degree line)

Concordance Correlation Coefficient (cont.)

- Consider the expected squared difference

$$E\left[(X - Y)^2\right] = \left(\mu_x - \mu_y\right)^2 + \left(\sigma_x^2 + \sigma_y^2 - 2\sigma_{xy}\right) =$$

$$\left(\mu_x - \mu_y\right)^2 + \left(\sigma_x - \sigma_y\right)^2 + 2(1 - \rho)\sigma_x\sigma_y$$

- If X and Y have perfect agreement, this is 0

Concordance Correlation Coefficient (cont.)

- To scale between -1 and 1

$$\rho_c = 1 - \frac{E\left[\left(X - Y\right)^2\right]}{\sigma_x^2 + \sigma_y^2 + \left(\mu_x - \mu_y\right)^2}$$

Concordance Correlation Coefficient (cont.)

- The sample estimate is

$$\hat{\rho}_c = \frac{2s_{xy}}{s_x^2 + s_y^2 + \left(\bar{x} - \bar{y}\right)^2}$$

where

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \qquad s_x^2 = \frac{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)^2}{n} \qquad s_{xy} = \frac{\sum_{i=1}^{n} \left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{n}$$

Concordance Correlation Coefficient (cont.)

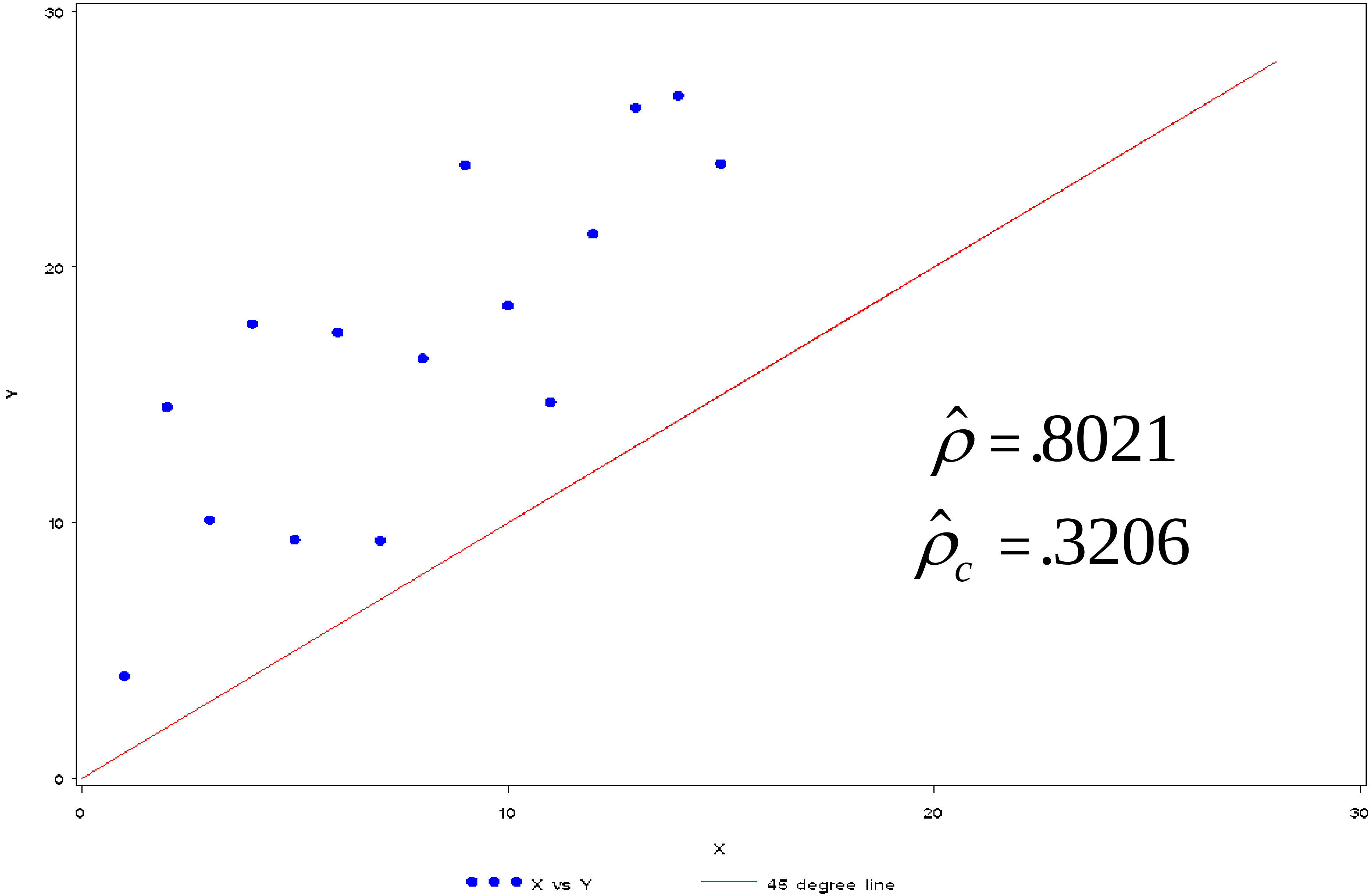
- Properties
  - $-1 \leq -|r| \leq r_c \leq |r| < 1$
  - $r_c = 0$ if and only if $r = 0$
  - $r_c = r$ if and only if $s_x = s_y$ and $m_x = m_y$
  - $r_c = \pm 1$ if and only if $r = \pm 1$ and $s_x = s_y$ and $m_x = m_y$
    - I.e. each set of (X_i, Y_i) is in perfect positive or negative agreement
      - { (1,1), (2,2), (3,3), (4,4), (5,5) }, or { (5,1), (4,2), (3,3), (2,4), (1,5) }

Macro

- %linscon(inds=dataset, x=xvar, y=yvar, labelX='X', labelY='Y', labelLegend='X vs Y', gtitle="Lin's Concordance vs Pearson Correlation");

- Also specify ***nboot***, the number of bootstrap replications (for confidence intervals if $n < 30$)

Lin's Concordance vs Pearson Correlation



Pearson Product Moment Correlation Coefficient = 0.8021

Lin s Concordance Correlation Coefficient = 0.3205, Bootstrap 95% Confidence Interval: (0.0000, 0.5454)

Summary

- assessing agreement is important
- r_c can be a more appropriate way to measure agreement compared to standard methods
- r_c has straightforward graphical interpretation
- linscon.sas macro is easy to use and freely available

References

- Lawrence, Lin. (1989). A Concordance Correlation Coefficient to Evaluate Reproducibility. *Biometrics, Vol 45, Mar. 1989, pp. 255-268*

- Barnhart, Huiman, et. al. (2002). Overall Concordance Correlation Coefficient for Evaluating Agreement Among Multiple Observers. *Biometrics, Vol 58, Dec. 2002, pp. 1020-1027*

- Cheng, Nancy (2004), linscon.sas, http://cando.ucsf.edu/resources/software/linscon.sas.txt

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