96 :)

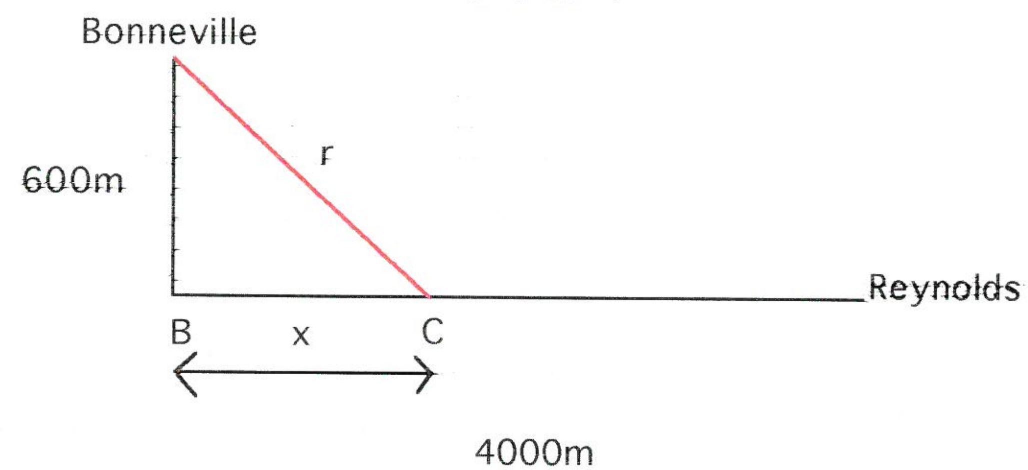
3/10/98

Dear Bonneville Engineering staff,

Upon being hired to study the problem of total construction cost for the new power line additions, I have came up with some solutions that this report will cover in great detail for you.

The first thing I was asked to do is determine the minimum cost. Before even coming up with an equation that models the cost, we first need to understand the relationship between distances. I direct your attention to Picture 1.

PICTURE 1



In this picture we can see the physical world relationships that we are trying to model. Bonneville is located at the top of the picture, and Reynolds on the lower right hand corner. They are separated by roughly 600 meters of the Columbia river. The distance that we are looking to obtain is the red line, r.

? Randold.

We already know from the budget reports of 1987 that to lay power lines underwater cost 4 times as much as laying them on land, mostly because of the danger and extra work involved. Since we know this relationship, and we know that to lay 1 meter of power line on the ground cost $20, we can get a rough equation that gives us the total cost for underwater line: $H2O\$ = 80 * \sqrt{x^2+600^2}$.

This equation is based on the Pythagorean Theorem, which states that in a right triangle, (which is the angle we are assuming that Bonneville makes with point B in Picture 1), $x^2+y^2=r^2$, where x and y are lengths of the 2 sides, and r is the length of the hypotenuse. So solving for r would give us $r= \sqrt{x^2+y^2}$. We don't know x yet, but we do know that y is 600 meters.

So now that we have our equation for the cost of underwater line, we need to find our equation for cost of ground line. This one is much easier. Our price per meter of ground line is $20, so we get an equation that looks like this: $Land\$ = 20(4000-x)$. This is because 4000-x represents the distance from point B that line r touches the other side of the river (at point C), minus an initial distance of 4000 meters from point B to Reynolds (4000-x is nothing more than the distance from point C to Reynolds). I urge your attention to Picture 1 again. ok :)

Now gentlemen, to get our equation for total cost, we simply just add both equations (H2O$ + Land$) together and get: $80 * \sqrt{x^2+600^2} + 20(4000-x)=C$, and this is our total cost for the job.

You may ask, "But how do we know what x is?", but I assure you, this will be answered shortly, as this is the most important part of the problem.

To find out what distance x needs to be to obtain the minimum cost, we need to do some math. The derivative of a function can tell us many things about the original function. Specifically what we are looking for is the x value which gives us the lowest y value, the minimum.

Below are the steps taken, with notes, to find the derivative of our cost function.

• $C= 80\sqrt{(x^2+600^2)}+20(4000-x),\ 0\le x\le 4000$

This is our cost function. Note that x has to be greater or equal to 0. This is because there is no such thing as a negative distance.

• $C' = [d/dx\ 80\sqrt{(x^2+600^2)}] + d/dx\ 20(4000-x)$

Here is how we set up our derivative.

• $[80 * d/dx\ (x^2+600^2)^{(1/2)} + (x^2+600^2)^{(1/2)} * d/dx\ 80] + [d/dx\ 20(4000-x)]$

not necessary, but workable

As this step shows, there are really several steps involved. Here we use the Product rule to determine the derivative of $80 * (\sqrt{x^2+600^2})$. We could also use a derivative rule called the Chain rule, but I prefer using the Product rule; either way, the result is the same.

• $[80 * x/(x^2+360000)^{(1/2)}] + [(360000+x^2)^{(1/2)} * 0] + [d/dx\ 80000-20x]$

Here we start to simplify things on the left side, but we still need to determine the derivative of 80000-20x.

•$[80x/(x^2+360000)^{(1/2)} + 0] + (-20)$

Now we have both derivatives, and we need to simplify things by adding both of them together to obtain a final equation for our derivative.

•$C' = 80x/(x^2+360000)^{(1/2)} - 20$

And this is the equation of our derivative.

To find the minimum path to take from Bonneville we need to set C', our derivative, to zero (which is the lowest y value possible, since we are only concerned with positive distances), and solve for x. Where the line of the derivative crosses the x axis on the graph, that is where the function equals 0, as shown in picture 2.

this doesn't really explain why this process works

Using algebraic operations, we obtain:

•$80x/(x^2+360000)^{(1/2)} - 20 = 0$

•$80x/(x^2+360000)^{(1/2)} = 20$

•$80x = 20 * (x^2+360000)^{(1/2)}$

•$80x/20 = (x^2+360000)^{(1/2)}$

•$4x = (x^2+360000)^{(1/2)}$

•$16x^2 = x^2+360000$

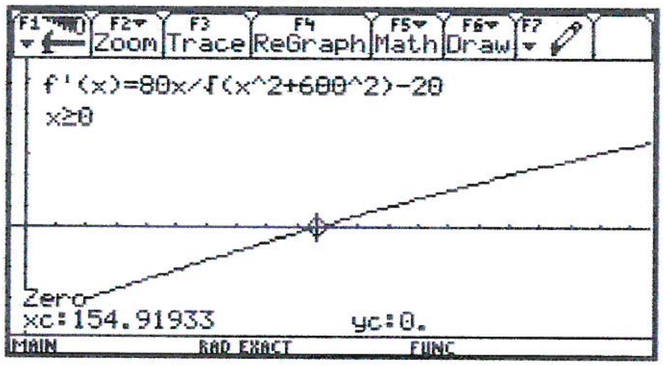
•$15x^2 = 360000$

•$x^2 = 2400$

•$x = 154.919$ meters

I direct your attention to Picture 2 for a graphical representation of how x was determined.

PICTURE 2

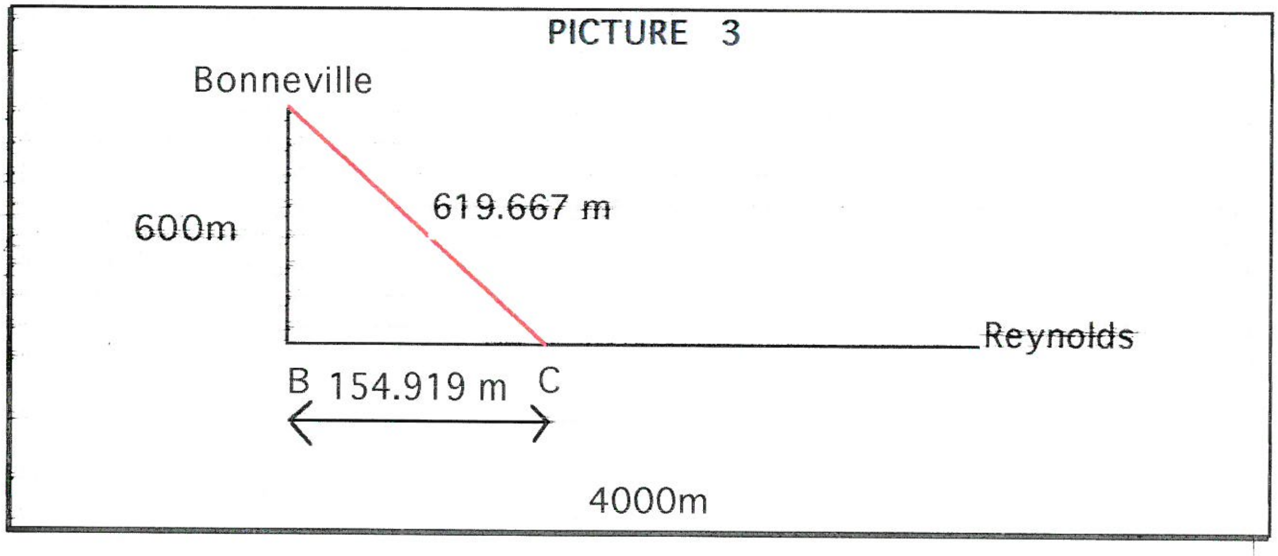


This work tells us that heading away from Bonneville, we need to find a point C that is 154.919 meters away from point B to minimize our total cost. So this means that we have calculated our x value! Now, using the Pythagorean Theorem to get the length of r, we get:

600^2+154.919^2=r^2

r=619.677 meters

Now we can fill in some more information on our graphical representation, to start forming a complete picture. (See Picture 3).



PICTURE 3

To calculate our total cost, we can now simply substitute in our values:
($80*619.677)+(20*(4000-154.919))
which gives us a total cost of $126474.98

However, since the amount that underwater line costs over the years varies with trends, I was asked to calculate this formula in $p as well. Since the cost for land line remains the same, we only have to adjust the cost equation where 80 (the cost of underwater line) was. Because the price of underwater line may vary over time, it isn't a fixed value. So instead of using $80 for the price of underwater line, we will use the variable p.

$C= p\sqrt{(x^2+600^2)}+20(4000-x),\ 0\le x\le 4000$

We still have the same cost formula, except notice how the price of the underwater line is replaced with the variable p.

•C'= [d/dx p√(600^2+x^2)] + [d/dx 20(4000-x)]

To find the minimum cost, we first need to find the derivative of our new cost function, just like before where we did this when p=$80.

•[p * d/dx (360000+x^2)^(1/2) + (360000+x^2)^(1/2) * d/dx p] + [d/dx 20(4000-x)]

•[p * x/(x^2+360000)^(1/2)] + [(360000+x^2)^(1/2) * 0] + [d/dx 80000-20x]

•[px/(x^2+360000)^(1/2) + 0] + (-20)

•px/(x^2+360000)^(1/2) - 20

To find the minimum path we need to set C' to zero and solve for x. Our computed value of X will tell us how many meters away from point B that r will need to extend to.

•px/(x^2+360000)^(1/2)-20= 0

•px/(x^2+360000)^(1/2)= 20

•(x^2+360000)^(1/2)= 20/px

•x(x^2+360000)^(1/2)= 20/p

- [x(x^2+360000)^(1/2)]^2= [20/p]^2

- p^2*x^2= 400x^2+12000^2

- (p^2-400)*x^2= 12000^2

- x^2= 12000^2/(p^2-400)

- x= 12000* 1/(p^2-400)^(1/2)

- x= 12000/(p^2-400)^(1/2) meters, p>$20 Good

Now we substitute our x back into the Pythagorean Theorem equation to obtain our r value, which is necessary in computing the total minimum cost. Our r value will tell us how long r is to obtain a minimum cost.

- [12000/(p^2-400)^(1/2)]^2+600^2= r^2

- 144000000/(p^2-400)+ 360000= r^2

- [144000000/(p^2-400)+ 360000= r^2]^(1/2)

- 600*[1/(p^2-400)]^(1/2) *p= r

- 600p/(p^2-400)^(1/2) meters= r

And this is our equation that gives us the length of r, which we will use to obtain a minimum cost for the job.

With this new information, we can now substitute x back into our cost function to obtain the minimum cost. Recall from above that our cost function is C= p√(x^2+600^2)+20(4000-x).

•p√(x^2+600^2)+20(4000-x)= C

•600p/(p^2-400)^(1/2)*p-240000/(p^2-400)^(1/2)+80000= C

•600p^2/(p^2-400)^(1/2)-240000/(p^2-400)^(1/2)+80000= C

•[600(p^2-400)^(1/2)]+80000= C

Where C is the total minimum cost for the job in dollars.

We can check this formula as well! For example, if we substitute 80 for p, which would resemble the current situation of underwater line prices, we get:

good sanity check

•[600(p^2-400)^(1/2)]+80000= C

•[600(80^2-400)^(1/2)]+80000= C

•[(6000)^(1/2)*600]+80000= C

•≈$126475

Which is what we obtained in our exploration of the minimum cost if underwater line is 4 times as expensive as the $20 land line. This proves that this function works.

In doing this project there are several major assumptions that we are making.

1. We are assuming that the triangle made from going to Bonneville, then point B, then point C is a right angle (90 degrees). It is because of this condition that we can use the Pythagorean Theorem.

2. We are also assuming that all given distances are correct. For example, it is very difficult to measure the width of a river exactly, mainly because the width of a river isn't uniform. Also the distance from Bonneville to Reynolds is difficult to measure precisely.

3. Lastly, we are also assuming that underwater line will always be more expensive than ground line ($20), which intuitively makes sense. For example, if underwater line were cheaper than 20$, then our second cost function would be undefined, since we can't take the square root of a negative number and obtain a real (non complex number) answer. Or course, if this situation ever arose we could simply substitute these values into our first cost function:

$p\sqrt{(x^2+600^2)}+q(4000-x),\ 0 \le x \le 4000$

Where p and q represent the costs of underwater and ground line.

Justin Smith/Calculus 251\Math Project/York

I believe that I have determined the best path to take to obtain the minimum cost for the job as requested. I leave the actual construction in the capable hands of you Bonneville engineers.

I hope you are pleased with my work, and as usual, if you have any questions, or are seeking future employment, feel free to contact me at the usual address. you're hiring? :)

Best wishes,

Justin Smith

This is an excellent paper!

It is obvious you took it seriously.

Explanations are clear and thorough, with 1 small exception.

Presentation is flawless.